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The Effect of Curvature Variation on the Scattering from Rough Surfaces

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The general high-frequency, rough-surface reflection process is treated by the method of stationary phase. In particular, the principle of stationary phase is applied to each of the local reflection events, which appear to depend on surface curvature as well as slope. However, we show that the number of reflecting highlights also depends on curvature and cancels the curvature dependence. These results agree fully with those obtained by Beckmann and Spizzichino for the special case of the Gaussian process.

KEY WORDS: Reflection of radiation; surface reflection; physical optics; stationary phase method; Huygen's principle; Gaussian surface.

1. INTRODUCTION

It has been shown by Beckmann and Spizzichino[(1)] and others that the statistical properties of electromagnetic waves reflected by Gaussian surfaces depend only upon the rms surface slope. They use a method based on the assumption that the surface appears nearly locally flat relative to the scale (wavelength) of the incident radiation [i.e., the wavelength is assumed to be small compared with a typical length (surface radius of curvature)]. We show here that their calculation is consistent with what we know about the local high-frequency reflection event. In particular, the single reflection event appears to depend on surface curvature as well as on slope; however, we show that the number of reflecting highlights also depends on curvature and this cancels the curvature dependence.

Consider a physical model for the process as follows: when the electromagnetic wevelength is small compared with the radius of curvature of the surface, the curvature determines the intensity of the return from a single reflecting point or highlight (the normal to the tangent plane bisects the angle between the incident and reflected

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rays). The scatter coefficient can be calculated directly from the Kirchhoff integral by the method of stationary phase.[2]

For a one-dimensional surface (one independent variable x determines the surface displacement), the scatter coefficient for backscattering at an angle of incidence θ from the vertical and for electrical field vector in the y direction is (see Beckmann and Spizzichino,[1] p. 23; here, we let their $-\theta_2 = \theta_1 \equiv \theta$)

$$\rho = -(1/2L \cos \theta) \int_{-L}^{L} R(x)\, \phi(x)\, e^{if(x)}\, dx \tag{1}$$

where $R(x)$ is the Fresnel reflection coefficient for a smooth, finitely conducting plane with incident wave horizontally polarized and angle of incidence the local angle of incidence $\phi(x) = \theta - \beta(x)$, where $\beta(x)$ is the local surface slope angle. The functions $R(x)$, $\phi(x)$, and $f(x)$ are given as follows:

$$R(x) = \{\cos \phi(x) - [Y^2 - \sin^2 \phi(x)]^{1/2}\}/\{\cos \phi(x) + [Y^2 - \sin^2 \phi(x)]^{1/2}\}$$

$$\phi(x) = \zeta'(x) \sin \theta + \cos \theta$$

$$f(x) = (4\pi/\lambda)\, x \sin \theta - (4\pi/\lambda)\, \zeta(x) \cos \theta$$

where Y is the normalized electrical admittance[1] of the medium below the surface and $\zeta(x)$ and $\zeta'(x)$ are surface height and slope, respectively. [The same formula (1) applies to acoustic reflection at a pressure release surface with R the acoustic reflection coefficient.] This definition then gives the ratio of the amplitude of the reflected electric field to that of the field specularly reflected from a perfectly conducting, flat surface of the same overall size with the same angle of incidence at the same distance, when the incident wave is horizontally polarized. (The results are easily extended to forward scattering with arbitrary reflection angles.)

When the variation of surface height is large compared to the electrical wavelength λ, the only contributions to the integral are those given at the "stationary phase" points $x = x_n$. These points are defined by $df(x)/dx = 0$. The amount of each of these contributions is[2]

$$-[R(2\pi)^{1/2}\, \phi(x_n)/|f''(x_n)|^{1/2}] \exp\{i[f(x_n) \pm \pi/4]\}$$

where the upper or lower sign is to be taken in the exponential according as $f''(x_n)$ is positive or negative, and at these points,

$$-R(x_n) = (Y - 1)/(Y + 1)$$

$$\phi(x_n) = \sec \theta$$

$$|f''(x_n)| = 2(2\pi/\lambda)^{1/2}\, |\zeta''(x_n)| \cos \theta$$

so that Eq. (1) becomes

$$\rho = \frac{-R\lambda^{1/2}}{2\, 2^{1/2} L \cos \theta} \sum_{n=1}^{N} r_n^{1/2} e^{i\psi_n} \tag{2}$$

where r_n is the radius of curvature, given by

$$r_n = 1/|\,\zeta''(x_n)|\cos^3\theta$$

and the angles ψ_n are defined by

$$\psi_n \equiv f(x_n) \pm \pi/4$$

with the stipulation on the sign that given above. Assuming incoherence of specular points ($\langle\psi_m\,\psi_n\rangle = 0$), then the mean-square scatter coefficient is

$$\langle|\,\rho\,|^2\rangle = (R^2\lambda/8L^2\cos^2\theta)\left\langle\sum_{n=1}^{N} r_n\right\rangle \tag{3}$$

For an ergodic surface, we have further that

$$\left\langle\sum_{n=1}^{N} r_n\right\rangle \to \langle N\rangle\langle r_n\rangle \tag{4}$$

asymptotically with $\langle N\rangle \to \infty$, where $\langle r_n\rangle$ is the mean radius of curvature at the reflecting points. A similar result has been obtained previously by Kodis[3] for the special case of a perfectly conducting surface.

Repeating the above procedure for a two-dimensional surface (two independent variables x and y for surface displacement, e.g., terrain or ocean surfaces), the scatter coefficient for backscatter at an angle of incidence θ from the vertical is (see Beckmann and Spizzichino,[1] p. 26; letting their $-\theta_2 \neq \theta_1 \equiv \theta$ and $\theta_3 = 0$)

$$\rho = (1/4xy\cos\theta)\int_{-y}^{y}\int_{-x}^{x} R(x, y)\,\phi(x, y)\,e^{if(x,y)}\,dx\,dy \tag{5}$$

$R(x, y)$ is in general not a Fresnel coefficient, except at the stationary phase points, where

$$-R(x_n, y_n) = (Y-1)/(Y+1)$$

as above. [For acoustic reflection from a pressure release surface, (5) holds at every point and $R(x, y) = R$, the reflection coefficient at the pressure release surface.] The functions $\phi(x, y)$ and $f(x, y)$ are given as

$$\phi(x, y) = \zeta_x(x, y)\sin\theta + \cos\theta$$

$$f(x, y) = (4\pi/\lambda)\,x\sin\theta - (4\pi/\lambda)\,\zeta(x, y)\cos\theta$$

where $\zeta_x(x, y) \equiv \partial\zeta(x, y)/\partial x$ and $\zeta(x, y)$ is the surface height. The stationary phase points for this integral are given by

$$\partial f(x, y)/\partial x = \partial f(x, y)/\partial y = 0$$

To evaluate the contribution at the nth stationary phase point, rotate the x, y coordinate system to coincide with the direction of principal radii of curvature at that point so that

$$\partial^2 f(x_n, y_n)/\partial x\,\partial y = 0$$

In the neighborhood of a particular stationary phase point, defining $\xi \equiv x - x_n$ and $\eta \equiv y - y_n$, the Taylor series expansion for f is

$$f(\xi, \eta) = f(x_n, y_n) + \tfrac{1}{2}\xi^2 f_{xx}(x_n, y_n) + \tfrac{1}{2}\eta^2 f_{yy}(x_n, y_n) + \cdots$$

since

$$f_x(x_n, y_n) = f_y(x_n, y_n) = f_{xy}(x_n, y_n) = 0$$

The contributions from each of these points is then[2]

$$\frac{-2\pi R\phi(x_n, y_n) \exp\{i[f(x_n, y_n) \pm (\pi/2 \text{ or } 0)]\}}{|f_{xx}(x_n, y_n) f_{yy}(x_n, y_n)|^{1/2}}$$

where the upper or lower sign is to be taken in the exponential according as $f_{xx}(x_n, y_n)$ and $f_{yy}(x_n, y_n)$ are both positive or negative, and zero is taken otherwise. Also,

$$\begin{aligned} \phi(x_n, y_n) &= \sec\theta \\ |f_{xx}(x_n, y_n)| &= 2(2\pi/\lambda)^{1/2} |\zeta_{xx}(x_n, y_n)| \cos\theta \\ |f_{xy}(x_n, y_n)| &= 2(2\pi/\lambda)^{1/2} |\zeta_{yy}(x_n, y_n)| \cos\theta \end{aligned}$$

so that Eq. (5) becomes

$$\rho = (-R\lambda/8xy\cos\theta) \sum_{n=1}^{N} (a_n b_n)^{1/2} \exp(i\psi_n') \tag{6}$$

where a_n and b_n are the principal radii of curvature at the nth stationary phase point. The inverse of the product $a_n b_n$ is the total curvature $K_n = 1/a_n b_n$, defined by[3]

$$K_n \equiv |\zeta_{xx}(x_n, y_n)\, \zeta_{yy}(x_n, y_n)| \cos^4\theta$$

and

$$\psi_n' \equiv f(x_n, y_n) \pm (\pi/2 \text{ or } 0)$$

with the stipulation on sign or 0 as above. For incoherent specular returns, the mean-square scatter coefficient is

$$\langle |\rho|^2 \rangle = (R^2\lambda^2/64x^2y^2\cos^2\theta) \left\langle \sum_{n=1}^{N} a_n b_n \right\rangle \tag{7}$$

and for the ergodic surface,

$$\left\langle \sum_{n=1}^{N} a_n b_n \right\rangle \to \langle N \rangle \langle a_n b_n \rangle \tag{8}$$

[3] Normally,[4] total curvature K is defined by $|\zeta_{xx}\zeta_{yy} - \zeta_{xy}^2| \cos^4\theta$, where $\cos^4\theta = (1 + \zeta_x^2 + \zeta_y^2)^{-2}$ or $\tan^2\theta = \zeta_x^2 + \zeta_y^2$. For the locally rotated coordinates, however, $\zeta_{xy}(x_n, y_n) = 0$ for each r giving the above definition (in the new coordinates).

asymptotically as $\langle N \rangle \to \infty$, where $\langle a_n b_n \rangle$ is the mean product of principal radii of curvature at the reflecting points. Kodis[3] obtained a similar result (restricted to perfectly conducting surfaces).

We show next that for statistically homogeneous surfaces having first-order joint probability densities of first and second directional derivatives of the surface, the mean random sums in (3) and (7) can be calculated in terms of these, and by so doing, one finds an annullment of surface curvature effects. The method is based on the differential (geometric) probability calculus associated more generally with other "crossing" problems as well. Using these methods, Longuet-Higgins[5] has studied the quantity $\langle N \rangle$. It will be shown here how these same methods can be used to evaluate the means of the random sums given above.

2. CALCULATIONS OF MEANS

Let us consider the mean random sum in (3) first. By dividing the interval of length $2L$ into k subintervals of length Δx each, then for the statistically homogeneous surface, by linearity, we have as a limit

$$\left\langle \sum_{n=1}^{N} r_n \right\rangle = \lim_{k\to\infty} k\langle rp \rangle \tag{9}$$

where, for each k, r is the radius of curvature in a given subinterval and p the probability (given r) that a specular point occurs in that subinterval, the expectation taken over r. The quantity $\langle rp \rangle$ approximates the mean random sum of radii at the specular points in the given subinterval, approaching the true value as $k \to \infty$. Then, $k\langle rp \rangle$ approximates the mean random sum over the entire interval. If the limit given by (9) exists, then it is a valid way of calculating the mean of this random sum. From the usual probability argument,[5] we have the stochastic integral (approximating p for Δx small) as

$$p = \int_{\Delta x} p(\zeta' \mid \zeta'')\, d\zeta'$$

where $\zeta' = \tan\theta$ and $p(\zeta' \mid \zeta'')$ is the probability density of slope conditioned on curvature. From the Jacobian $\mid \zeta'' \mid = 1/(r \cos^3\theta)$, this becomes

$$rp = (\Delta x/\cos^3\theta)\, p(\zeta' \mid \zeta'')$$

so that from (9), in the limit as $\Delta x \to 0$,

$$\left\langle \sum_{n=1}^{N} r_n \right\rangle = (2L/\cos^3\theta)\, p(\zeta') \tag{10}$$

where $p(\zeta')$ is the probability density of surface slope at any given point and $\zeta' = \tan\theta$. This result can also be derived from (4) to show explicitly the cancellation of curvature. Proceeding to do this, first calculate $\langle r_n \rangle$. For this, it is sufficient to condition on the occurrence of a specular point in a given subinterval Δx and let $\Delta x \to 0$. (It is necessary to take an interval, since conditioning on the occurrence at a point is overly restrictive and in so doing, one obtains a divergent result in most cases.) Let us first obtain the probability density function of curvature at a specular point given that it occurs in a given subinterval Δx. Consider the probability element

$$p(\zeta', \zeta'')\, d\zeta'\, d\zeta''$$

for the first two derivatives at a random position (in Δx) x (near $x_0 = 0$ without loss of generality). Transforming variables $\zeta' \to x$ and normalizing, this becomes

$$p(\zeta', \zeta'') \mid \zeta'' \mid dx\, d\zeta''/p(\Delta x)$$

where $p(\Delta x)$ is defined to be the probability that a specular point occurs in Δx, and for Δx small, is approximated by

$$p(\Delta x) = \langle N \rangle (\Delta x)/2L$$

Then, the joint probability density for a random point x and curvature ζ'' given that $0 \leqq x \leqq \Delta x$ and $\zeta' = \tan\theta$ is approximately given by

$$p_n(x, \zeta'') = 2Lp(\zeta', \zeta'') \mid \zeta'' \mid /(\langle N \rangle\, \Delta x)$$

Integrating x over Δx and letting $\Delta x \to 0$, then the desired probability density of curvature at a specular point is

$$p_n(\zeta'') = 2Lp(\zeta', \zeta'') \mid \zeta'' \mid / \langle N \rangle$$

where $\zeta' = \tan\theta$. From this,

$$\langle r_n \rangle = \int_{-\infty}^{\infty} r_n p_n(\zeta'')\, d\zeta''$$

and since $r_n = 1/(\mid \zeta'' \mid \cos^3\theta)$, then

$$\langle r_n \rangle = 2Lp(\zeta')/(\langle N \rangle \cos^3\theta)$$

so that

$$\left\langle \sum_{n=1}^{N} r_n \right\rangle = \langle N \rangle \langle r_n \rangle = 2Lp(\zeta')/\cos^3\theta$$

in agreement with (10). Thus, for the one-dimensional surface at least, the mean random sum of radii of curvature at the reflecting points is in fact independent of curvature and only depends upon the probability density of surface slope.

Extending the above arguments to the two-dimensional surface case, divide the area $4xy$ into k^2 units of area $\Delta x\, \Delta y$ each ($\Delta x = \Delta y$). Then,

$$\left\langle \sum_{n=1}^{N} a_n b_n \right\rangle = \lim_{k\to\infty} k^2 \langle abp \rangle \tag{11}$$

where, for each k, a and b are the principal radii of curvature in a given unit and p is the probability (given a and b) that a specular point occurs in that unit, the expectation taken jointly over a and b. As before,

$$p = \int_{\Delta x} \int_{\Delta y} p(\zeta_x, \zeta_y \mid \zeta_{xx}, \zeta_{xy}, \zeta_{yy})\, d\zeta_x(x)\, d\zeta_y(y)$$

where $\zeta_y = \tan\theta$ and $\zeta = 0$ and $p(\zeta_x, \zeta_y \mid \zeta_{xx}, \zeta_{xy}, \zeta_{yy})$ is the joint probability density of slopes [directional derivatives $\xi_x \equiv \partial\zeta(x, y)/\partial x$, $\zeta_y \equiv \partial\zeta(x, y)/\partial y$] conditioned on curvatures [$\zeta_{xx} \equiv \partial^2\zeta(x, y)/\partial x\, \partial x$, $\zeta_{xy} \equiv \partial^2\zeta(x, y)/\partial x\, \partial y$, and $\zeta_{yy} \equiv \partial^2\zeta(x, y)/\partial y\, \partial y$]. From the Jacobian

$$J = \mid \zeta_{xx}\zeta_{yy} - \zeta_{xy}^2 \mid = 1/ab \cos^4\theta$$

we have

$$abp(ab) = (\Delta x\, \Delta y/\cos^4\theta)\, p(\zeta_x, \zeta_y \mid \zeta_{xx}, \zeta_{xy}, \zeta_{yy})$$

so that

$$\left\langle \sum_{n=1}^{N} a_n b_n \right\rangle = (4xy/\cos^4\theta)\, p(\zeta_x, \zeta_y) \tag{12}$$

where $p(\zeta_x, \zeta_y)$ is the joint probability density of slopes at a given point and $\zeta_x = \tan\theta$ and $\zeta_y = 0$. This result is also obtained, starting with (8), from $\langle a_n b_n \rangle$ as follows: the probability element

$$p(\zeta_x, \zeta_y, \zeta_{xx}, \zeta_{xy}, \zeta_{yy})\, d\zeta_x\, d\zeta_y\, d\zeta_{xx}\, d\zeta_{xy}\, d\zeta_{yy}$$

for the first and second derivatives of the surface at a random position [in $(\Delta x, \Delta y)$] (x, y) near (x_0, y_0), $x_0 = y_0 = 0$ without loss of generality, is transformed and normalized as follows:

$$[p(\zeta_x, \zeta_y, \zeta_{xx}, \zeta_{xy}, \zeta_{yy})J/p(\Delta x, \Delta y)]\, dx\, dy\, d\zeta_{xx}\, d\zeta_{xy}\, d\zeta_{yy}$$

where $J = \mid \zeta_{xx}\zeta_{yy} - \zeta_{xy}^2 \mid$ and $p(\Delta x, \Delta y)$ is the probability that a specular point occurs in $(\Delta x, \Delta y)$, and is approximated by

$$p(\Delta x, \Delta y) = \langle N \rangle (\Delta x \Delta y)/4xy$$

The joint probability density of a random point (x, y) and curvature $(\zeta_{xx}, \zeta_{xy}, \zeta_{yy})$, given that $0 \leqq x, y \leqq \Delta x, \Delta y$; $\zeta_x = \tan\theta$, and $\zeta_y = 0$, is approximately

$$4xyp(\zeta_x, \zeta_y, \zeta_{xx}, ,\zeta_{xy}, \zeta_{yy})J/(\langle N \rangle\, \Delta x\, \Delta y)$$

Integrating x over Δx and y over Δy, then the desired probability density of curvature at a specular point for $\Delta x = \Delta y \to 0$ is

$$p_n(\zeta_{xx}, \zeta_{xy}, \zeta_{yy}) = 4xyp(\zeta_x, \zeta_y, \zeta_{xx}, \zeta_{xy}, \zeta_{yy})J/\langle N \rangle$$

where $\zeta_x = \tan\theta$, $\zeta_y = 0$, and $J = \mid \zeta_{xx}\zeta_{yy} - \zeta_{xy}^2 \mid = 1/(a_n b_n \cos^4\theta)$. From this,

$$\langle a_n b_n \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a_n b_n p_n(\zeta_{xx}, \zeta_{xy}, \zeta_{yy})\, d\zeta_{xx}\, d\zeta_{xy}\, d\zeta_{yy}$$

giving

$$\langle a_n b_n \rangle = 4xyp(\zeta_x, \zeta_y)/(\langle N \rangle \cos^4 \theta)$$

so that

$$\left\langle \sum_{n=1}^{N} a_n b_n \right\rangle = \langle N \rangle \langle a_n b_n \rangle = 4xyp(\zeta_x, \zeta_y)/\cos^4 \theta$$

in agreement with (12) above. Thus, we have a similar conclusion for the two-dimensional case as for the one-dimensional case and combining these results, (10) and (12), with (3) and (7), then the mean-square scatter coefficients are expressible as

$$\langle | \rho |^2 \rangle_1 = R^2 \lambda p(\zeta')/(4L \cos^5 \theta) \tag{13}$$

$$\langle | \rho |^2 \rangle_2 = R^2 \lambda^2 p(\zeta_x, \zeta_y)/(16xy \cos^6 \theta) \tag{14}$$

where $\zeta' = \zeta_x = \tan \theta$ and $\xi = 0$ for the one- and two-dimensional surfaces, respectively.

3. DISCUSSION

These formulas, (13) and (14), are what one might expect as the limiting case of the discrete, flat-facet surface model of geometric optics when the number of facets per unit area is increased while the size of each diminishes to zero. However, the derivation here of these results is based on the Kirchhoff model or "physical optical" model with the surface curvature taken into account. By the application of "stationary phase" arguments and differential (geometric) probability calculations, it has been possible to show exactly how it is that the curvature effects are not present (or are accounted for in the probability density of slope) in the final result. These results have been checked with those which are known for particular cases such as sinusoidal profiles and Gaussian surfaces(1) (two-dimensional Gaussian processes) and have been found to be in full agreement. This is illustrated by the following two examples.

4. EXAMPLES

Consider first the sinusoidal profile

$$\zeta(x) = A \sin[\omega(x - x_0)]$$

where A (or ω^{-1}) has slight variations (of order λ, so that this surface is noncoherently diffracting) and x_0 is a uniform random variable, $0 \leqq \omega x_0 \leqq 2\pi$ (so that this surface also is statistically homogeneous). We will show here that the use of either formula (4) with $\langle N \rangle = N$ and $\langle r_n \rangle = r_n$ (since these are fixed for given values of θ, L, A, and ω) or (10) gives the same result. To find $p(\zeta')$ in (10), take the first two derivatives

$$\zeta'(x) = A\omega \cos[\omega(x - x_0)]$$

and

$$\zeta''(x) = -A\omega^2 \sin[\omega(x - x_0)]$$

Then, ζ' and $-(\zeta''/\omega)$ have the circular uniform joint probability density (in polar form)

$$p_1(r, \alpha) = \delta(r - A\omega)/2\pi$$

where

$$\zeta' = r \cos \alpha, \qquad -(\zeta''/\omega) = r \sin \alpha$$

and $\delta(r - A\omega)$ is the Dirac delta function. In rectangular form, then,

$$p(\zeta', \zeta'') = \delta(r - A\omega)/2\pi r\omega$$

where

$$r^2 = (\zeta')^2 + (\zeta''/\omega)^2$$

Thus,

$$p(\zeta') = \int_{-\infty}^{\infty} [\delta(r - A\omega)/2\pi r\omega] \, d\zeta'' = 1/(\pi A\omega \sin \alpha)$$

so that

$$\left\langle \sum_{n=1}^{N} r_n \right\rangle = 2L/(\pi A\omega \sin \alpha \cos^3 \theta)$$

From (4) with $\langle N \rangle = N$ and $\langle r_n \rangle = r_n$, where simple calculation gives

$$N = 2 \frac{2L}{(2\pi/\omega)}$$

and

$$r_n = 1/(| \zeta_n'' | \cos^3 \theta) = 1/(A\omega^2 \sin \alpha \cos^3 \theta)$$

so that

$$Nr_n = 2L/(\pi A\omega \sin \alpha \cos^3 \theta)$$

in agreement with that found by the use of (10).

Consider as the next example the two-dimensional Gaussian surface (surface height given as a two-dimensional Gaussian process). For simplicity, consider the isotropic case where

$$p(\zeta_x, \zeta_y) = p(\zeta_x)\, p(\zeta_y)$$
$$p(\zeta_x, 0) = p^2(0) \exp(-\zeta_x^2/2\sigma^2)$$
$$p(0) = 1/(2\pi)^{1/2}\sigma$$

where σ is the rms surface slope. Using (14), then the mean-square scatter coefficient for this case is found to be

$$\langle |\rho|^2 \rangle_{\text{iso-Gauss}} = \frac{R^2\lambda^2 \exp[-(\tan^2\theta)/2\sigma^2]}{16\pi\sigma^2 A \cos^6\theta}$$

where A is the surface area. This form is in agreement with that found by Beckmann and Spizzichino.[1]

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